GAS MIXTURE FLOW IN A CYLINDRICAL CHANNEL AT INTERMEDIATE KNUDSEN NUMBERS

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A technique employing momentum equations averaged over channel section is used to obtain the kinetic coefficients describing nonisothermal flow of a gas mixture in a capillary at intermediate Knudsen number (Kn ≤ 0.25).

Analysis of gas mixture flow in a cylindrical channel at arbitrary values of Knudsen number is usually based on use of a linearized kinetic equation with model BKG collision integral in the Hamel–Ochugi form [1, 2]. The constraints of this model do not permit consideration of thermodiffusion effects, nor does the model provide sufficient accuracy for determining such quantities as the thermal and diffusion slip coefficients, even in the limit of small Knudsen numbers [3]. In [4, 5] a method was developed for calculation of kinetic coefficients for a nonisothermal mixture flow in a channel, based on averaging over the channel section a system of moment equations obtained by linearizing the kinetic equation with exact Boltzmann collision integral. In [6] this method was used to obtain expressions for a Poiseuille flow, thermocreep flow, and heat flow in a pure gas in a cylindrical channel. The results are valid over the range 0 < Kn ≤ 0.25. In the present study the corresponding kinetic coefficients characterizing nonisothermal flow of a gas mixture in a channel will be calculated.

We will consider the slow flow of a gas mixture in a circular capillary of radius R under the action of relative partial pressure gradient $k_\alpha = p_{\alpha 0}^{-1} dp_\alpha/dz$ and relative temperature gradient $\tau = T_0^{-1} dT/dz$. At low values of these gradients we may seek the distribution function in the form

$$f_\alpha(\mathbf{v}_\alpha, r, z) = f_{\alpha 0}\left[1 + k_\alpha z + \tau z\left(\beta_\alpha v_\alpha^2 - \frac{5}{2}\right) + \Phi_\alpha(\mathbf{v}_\alpha, r)\right],$$
$$f_{\alpha 0} = n_{\alpha 0}(\beta_\alpha/\pi)^{3/2}\exp(-\beta_\alpha v_\alpha^2),\ \beta_\alpha = m_\alpha/2kT_0, \tag{1}$$

where the nonequilibrium addition to the distribution function Φ_α is determined from the linearized Boltzmann kinetic equation [7]

$$v_{\alpha r}\frac{\partial \Phi_\alpha}{\partial r} + v_{\alpha z}k_\alpha + v_{\alpha z}\tau\left(\beta_\alpha v_\alpha^2 - \frac{5}{2}\right) + \frac{v_{\alpha\varphi}^2}{r}\frac{\partial \Phi_\alpha}{\partial v_{\alpha r}} - \frac{v_{\alpha r}v_{\alpha\varphi}}{r}\frac{\partial \Phi_\alpha}{\partial v_{\alpha\varphi}} = \sum_\beta \hat{L}_{\alpha\beta}\Phi_\alpha. \tag{2}$$

Multiplying Eq. (2) successively by $\Psi_\alpha(\mathbf{c}_\alpha)\exp(-c_\alpha^2)$, where $\Psi_\alpha(\mathbf{c}_\alpha) = c_{\alpha i};\ c_{\alpha i}c_{\alpha j} - \frac{1}{3}c_\alpha^2\delta_{ij};\ c_{\alpha i}\left(c_\alpha^2 - \frac{5}{2}\right)$ and $c_{\alpha i}c_{\alpha j}c_{\alpha k} - \frac{1}{5}c_\alpha^2(c_{\alpha i}\delta_{jk} + c_{\alpha j}\delta_{ik} + c_{\alpha k}\delta_{ij})$, $\mathbf{c}_\alpha = \beta_\alpha^{1/2}\mathbf{v}_\alpha$, then integrating over velocities, in the case of cylindrical problem geometry we arrive at moment equations of the form

$$\frac{1}{r}\frac{\partial}{\partial r}rp_{\alpha rz} + p_{\alpha 0}k_\alpha = -\sum_\beta\left[\frac{n_{\alpha 0}n_{\beta 0}kT_0}{n_0[D_{\alpha\beta}]_1}(u_{\alpha z} - u_{\beta z}) + \xi_{\alpha\beta}\left(\frac{h_{\alpha z}}{m_\alpha n_{\alpha 0}} - \frac{h_{\beta z}}{m_\beta n_{\beta 0}}\right)\right], \tag{3}$$

$$\frac{\partial}{\partial r}\left(m_\alpha s_{\alpha zrr} + \frac{2}{5}h_{\alpha z} + p_{\alpha 0}u_{\alpha z}\right) + \frac{m_\alpha}{r}(s_{\alpha zrr} - s_{\alpha z\varphi\varphi}) = -p_0^2\sum_\beta a_{\alpha\beta}p_{\beta rz}/p_{\beta 0}, \tag{4}$$

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$$\frac{2}{5r}\frac{\partial}{\partial r}r\left(\Pi_{\alpha zrrr}+\Pi_{\alpha zr\varphi\varphi}+\Pi_{\alpha zrzz}-\frac{5}{2}\,p_{\alpha rz}\right)+p_{\alpha 0}\tau=$$
$$=-\frac{kT_0}{m_\alpha}\sum_\beta \xi_{\alpha\beta}(u_{\alpha z}-u_{\beta z})-p_0^2T_0^{-1}\sum_\beta b_{\alpha\beta}h_{\beta z}/p_{\beta 0}, \quad (5)$$

$$\frac{1}{r}\frac{\partial}{\partial r}r(4\Pi_{\alpha zrrr}-\Pi_{\alpha zr\varphi\varphi}-\Pi_{\alpha zrzz})-\frac{10}{r}\Pi_{\alpha zr\varphi\varphi}=-\frac{25}{4}\frac{p_0^2}{T_0}\sum_\beta d_{\alpha\beta}m_\beta s_{\beta zrr}/p_{\beta 0}, \quad (6)$$

$$\frac{1}{r}\frac{\partial}{\partial r}r(4\Pi_{\alpha zr\varphi\varphi}-\Pi_{\alpha zrrr}-\Pi_{\alpha zrzz})+\frac{10}{r}\Pi_{\alpha zr\varphi\varphi}=-\frac{25}{4}\frac{p_0^2}{T_0}\sum_\beta d_{\alpha\beta}m_\beta s_{\beta z\varphi\varphi}/p_{\beta 0}, \quad (7)$$

$$s_{\alpha zrr}+s_{\alpha z\varphi\varphi}+s_{\alpha zzz}=0. \quad (8)$$

General expressions for the moments of $u_{\alpha z}$, $p_{\alpha rz}$, $h_{\alpha z}$, $s_{\alpha ijk}$, and $\Pi_{\alpha ijkl}$, defined on the nonequilibrium portion of the distribution function, are presented in [4]. In Eqs. (3)-(7) $[D_{\alpha\beta}]_1$ corresponds to the first approximation to the mutual diffusion coefficient of a binary mixture of α- and β-molecules [8], while the coefficients $\xi_{\alpha\beta}$, $a_{\alpha\beta}$, and $b_{\alpha\beta}$ were calculated in [9], and $d_{\alpha\beta}$ in [4].

Far from the walls this system of equations corresponds to the well-known 20-moment Grad approximation to the distribution function [10], which when linearized relative to small values of the dimensionless moments leads to the result

$$\Phi_\alpha^{as}(c_\alpha, r)=2\beta_\alpha^{1/2}(u_z^{as}+w_{\alpha z}^{as})c_{\alpha z}+2p_{\alpha 0}^{-1}p_{\alpha rz}^{as}c_{\alpha r}c_{\alpha z}+$$
$$+\frac{4}{5}\beta_\alpha^{1/2}p_{\alpha 0}^{-1}h_{\alpha z}^{as}c_{\alpha z}\left(c_\alpha^2-\frac{5}{2}\right)+2m_\alpha p_{\alpha 0}^{-1}\beta_\alpha^{1/2}c_{\alpha z}\left(s_{\alpha zrr}^{as}c_{\alpha r}^2+s_{\alpha z\varphi\varphi}^{as}c_{\alpha\varphi}^2+\frac{1}{3}s_{\alpha zzz}^{as}c_{\alpha z}^2\right), \quad (9)$$

where the superscript as denotes asymptotic values of the corresponding quantity (i.e., values outside the Knudsen layer). For a binary mixture, the expressions for these asymptotic values appearing in Eq. (9) have the form

$$w_{\alpha z}^{as}=(-1)^\alpha\frac{[D_{\alpha\beta}]_2\rho_{\beta 0}}{\rho_0 y_\alpha y_\beta}\left(\frac{d}{dz}y_1+y_\alpha y_\beta[\alpha_p]_2\frac{1}{p_0}\frac{dp}{dz}+y_\alpha y_\beta[\alpha_T]_1\frac{1}{T_0}\frac{dT}{dz}\right),\ \beta\neq\alpha; \quad (10)$$
$$p_{\alpha rz}^{as}(r)=-\frac{r}{2}\frac{\eta_\alpha}{\eta}\frac{dp}{dz};\ h_{\alpha z}^{as}=-\frac{5}{2}\frac{a_\alpha p_0}{y_\beta}\left[D_{\alpha\beta}\right]_2\frac{d}{dz}y_1+\frac{2T_0y_\alpha}{5|b|p_0}\alpha'_{p\alpha}\frac{dp}{dz}-\lambda'_\alpha\frac{dT}{dz},\ \beta\neq\alpha;$$
$$s_{\alpha zrr}^{as}=\frac{16}{25}\frac{T_0}{p_0}\frac{y_\alpha}{m_\alpha}\delta_\alpha\frac{dp}{dz};\ s_{\alpha z\varphi\varphi}^{as}=s_{\alpha zrr}^{as};\ s_{\alpha zzz}^{as}=-2s_{\alpha zrr}^{as}.$$

Expressions for the coefficients η_α, $[D_{\alpha\beta}]_2$, $[\alpha_p]_2$, and $[\alpha_T]_1$ can be found in [9], while those for $\alpha'_{p\alpha}$, λ'_α, and δ_α are given in [4]. Summing Eq. (3) over α and integrating over r, we arrive at an expression for the full viscous stress tensor P_{rz}, valid over the entire flow region:

$$p_{rz}(r)=\sum_\alpha p_{\alpha rz}(r)=-\frac{r}{2}\frac{dp}{dz}. \quad (11)$$

The values of $p_{\alpha rz}(r)$ are found by solution of Eq. (4). Substituting these values in Eq. (11) and integrating the expression obtained over r, we obtain

$$p_0^{-1}\sum_\alpha \eta_\alpha y_\alpha^{-1}\Big\{m_\alpha[s_{\alpha zrr}(R)-s_{\alpha zrr}(r)]+\frac{2}{5}[h_{\alpha z}(R)-$$
$$-h_{\alpha z}(r)]+p_{\alpha 0}[u_{\alpha z}(R)-u_{\alpha z}(r)]+\int_r^R\frac{m_\alpha}{r'}[s_{\alpha zrr}(r')-s_{\alpha z\varphi\varphi}(r')]\,dr'\Big\}=\frac{1}{4}(R^2-r^2)\frac{dp}{dz}. \quad (12)$$

We will now obtain averaged expressions for the reduced thermal flux, the difference between the flow component velocities, and the mean molar mixture velocity. Averaging Eqs. (3), (5)-(7), and (12) over channel section, we have

$$-\sum_{\beta}\left[\frac{n_{\alpha 0}n_{\beta 0}kT_0}{n_0[D_{\alpha\beta}]_1}(\langle u_{\alpha z}\rangle-\langle u_{\beta z}\rangle)+\xi_{\alpha\beta}\left(\frac{\langle h_{\alpha z}\rangle}{m_\alpha n_{\alpha 0}}-\frac{\langle h_{\beta z}\rangle}{m_\beta n_{\beta 0}}\right)\right]=L_{\alpha 1}, \tag{13}$$

$$-\frac{kT_0}{m_\alpha}\sum_{\beta}\xi_{\alpha\beta}(\langle u_{\alpha z}\rangle-\langle u_{\beta z}\rangle)-\frac{p_0^2}{T_0}\sum_{\beta}b_{\alpha\beta}\langle h_{\beta z}\rangle/p_{\beta 0}=L_{\alpha 2}, \tag{14}$$

$$-\frac{5}{4}\frac{p_0^2}{T_0}\sum_{\beta}d_{\alpha\beta}m_\beta(\langle s_{\beta zrr}\rangle+\langle s_{\beta z\varphi\varphi}\rangle)/p_{\beta 0}=L_{\alpha 3}, \tag{15}$$

$$p_0^{-1}\sum_{\alpha}\eta_\alpha y_\alpha^{-1}\left[\frac{m_\alpha}{2}(\langle s_{\alpha zrr}\rangle+\langle s_{\alpha z\varphi\varphi}\rangle)+\frac{2}{5}\langle h_{\alpha z}\rangle+p_{\alpha 0}\langle u_{\alpha z}\rangle\right]=L_4, \tag{16}$$

where

$$L_{\alpha 1}=\frac{2}{R}p_{\alpha rz}(R)+p_{\alpha 0}k_\alpha,$$

$$L_{\alpha 2}=\frac{4}{5R}\left[\Pi_{\alpha zrrr}(R)+\Pi_{\alpha zr\varphi\varphi}(R)+\Pi_{\alpha zrzz}(R)-\frac{5}{2}p_{\alpha rz}(R)\right]+p_{\alpha 0}\tau,$$

$$L_{\alpha 3}=\frac{2}{5R}[3\Pi_{\alpha zrrr}(R)+3\Pi_{\alpha zr\varphi\varphi}(R)-2\Pi_{\alpha zrzz}(R)],$$

$$L_4=-\frac{R^2}{8}\frac{dp}{dz}+p_0^{-1}\sum_{\alpha}\eta_\alpha y_\alpha^{-1}\left[m_\alpha s_{\alpha zrr}(R)+\frac{2}{5}h_{\alpha z}(R)+p_{\alpha 0}u_{\alpha z}(R)\right],$$

$$\langle Q_\alpha\rangle=\frac{2}{R^2}\int_0^R Q_\alpha(r)\,rdr.$$

In particular, for a binary mixture, solution of system (13)-(16) gives

$$\langle u_{1z}\rangle-\langle u_{2z}\rangle=-\frac{[D_{12}]_2}{p_0 y_1 y_2}\left[L_{11}+\frac{5}{2}(L_{12}a_1+L_{22}a_2)\right], \tag{17}$$

$$\langle u_{mz}\rangle=-L_{11}[D_{12}]_2[\alpha_p]_2p_0^{-1}+\frac{2T_0}{5p_0^2|b|}(L_{12}\alpha'_{p1}+L_{22}\alpha'_{p2})+\frac{2T_0}{5p_0^2}(L_{13}\delta_1+L_{23}\delta_2)+L_4\eta^{-1}, \tag{18}$$

$$\langle h_z\rangle=-[L_{11}[D_{12}]_2[\alpha_T]_1+\frac{T_0}{p_0y_1y_2}(L_{12}\lambda'_1y_2+L_{22}\lambda'_2y_1)\Big]. \tag{19}$$

To proceed further, it is necessary to find values of the unknown moments on the channel walls, appearing in the expressions for L_{ik}. As in [4, 6], we will use Loyalka's [11] approximate method.

We introduce distribution functions for incident and reflected molecules, so that $\Phi_\alpha=\Phi_\alpha^+$ for $c_{\alpha r}>0$ and $\Phi_\alpha=\Phi_\alpha^-$ for $c_{\alpha r}<0$. Using the conventional Maxwell condition for molecule reflection on the wall for the functions $\Phi_\alpha^\pm$ at $r=R$, we have

$$\Phi_\alpha^+(\mathbf{c}_\alpha,R)=2\beta_\alpha^{1/2}c_{\alpha z}(a+\overset{\text{as}}{w}_{\alpha z})+2p_{\alpha 0}^{-1}\overset{\text{as}}{p}_{\alpha rz}(R)c_{\alpha r}c_{\alpha z}+$$
$$+\frac{4}{5}\beta_\alpha^{1/2}p_{\alpha 0}^{-1}\overset{\text{as}}{h}_{\alpha z}c_{\alpha z}\left(c_\alpha^2-\frac{5}{2}\right)+2m_\alpha p_{\alpha 0}^{-1}\beta_\alpha^{1/2}c_{\alpha z}\left(\overset{\text{as}}{s}_{\alpha zrr}c_{\alpha r}^2+\overset{\text{as}}{s}_{\alpha z\varphi\varphi}c_{\alpha\varphi}^2+\frac{1}{3}\overset{\text{as}}{s}_{\alpha zzz}c_{\alpha z}^2\right),\ c_{\alpha r}>0; \tag{20}$$

$$\Phi_\alpha^-(\mathbf{c}_\alpha,R)=(1-\varkappa_\alpha)\Phi_\alpha^+(-c_{\alpha r},c_{\alpha\varphi},c_{\alpha z},R),\ c_{\alpha r}<0,$$

where $\varkappa_\alpha$ is the fraction of molecules experiencing diffuse reflection on the wall.

Using Eq. (9) and the definition of $p_{\alpha rz}$, Eq. (1.9) of [4], on the channel wall, after calculating the corresponding integrals with consideration of Eq. (20), we find the constant a:

$$a = -\left(\sum_{\alpha} \varkappa_\alpha \beta_\alpha^{1/2} p_{\alpha 0}\right)^{-1} \left[\frac{\sqrt{\pi} R}{4\eta} \sum_{\alpha} \eta_\alpha (2 - \varkappa_\alpha) \frac{dp}{dz} + \right.$$

$$\left. + \sum_{\alpha} \varkappa_\alpha \beta_\alpha^{1/2} \left(p_{\alpha 0} w_{\alpha z}^{\mathrm{as}} + \frac{1}{5} h_{\alpha z}^{\mathrm{as}} + \frac{m_\alpha}{2} s_{\alpha z r r}^{\mathrm{as}}\right)\right]. \tag{21}$$

If we employ Eq. (10), the quantity a is then expressed in terms of the concentration, pressure, and temperature gradients. Substituting this value in Eq. (20), we find the unknown quantities on the channel wall appearing in L_{ik}.

According to the thermodynamics of irreversible systems [12], the relationship between flows and gradients can be expressed in the form

$$\begin{pmatrix} \langle h_z \rangle \\ \langle u_{1z} \rangle - \langle u_{2z} \rangle \\ \langle u_{mz} \rangle \end{pmatrix} = -\begin{pmatrix} \Lambda_{qq}\Lambda_{q1}\Lambda_{qm} \\ \Lambda_{1q}\Lambda_{11}\Lambda_{1m} \\ \Lambda_{mq}\Lambda_{m1}\Lambda_{mm} \end{pmatrix} \begin{pmatrix} T^{-2}dT/dz \\ pT^{-1}dy_1/dz \\ T^{-1}dp/dz \end{pmatrix}, \tag{22}$$

where $\langle h_z \rangle = \langle q_z \rangle - \frac{5}{2}\sum_{\alpha} p_\alpha \langle w_{\alpha z} \rangle$ is the reduced mixture thermal flux and $\langle u_{mz} \rangle = \sum_{\alpha} y_\alpha \langle u_{\alpha z} \rangle$ is the mean-molecular mixture velocity. After the values of L_{ik} are calculated and written as linear combinations of the gradients of the corresponding thermodynamic quantities, comparison of Eq. (22) with Eqs. (17)-(19) will permit establishment of the explicit form of the expressions for the coefficients Λ_{ik}. These coefficients were obtained for a planar channel in [4]. In the case of a cylindrical channel, the values of Λ_{ik} coincide with their values for a planar channel with a simple replacement of the channel width d by the radius R, and replacement of the quantity δ_α by $(3/8)\delta_\alpha$. Some difference occurs in the expression for Λ_{mm}, which describes isothermal Poiseuille flow of the mixture in the channel. In this case the first term has the form $T_0R^2/8\eta$ instead of $T_0d^2/12\eta$. In addition, in the sixth order term for Λ_{mm} it is necessary to replace the term in parentheses $(11 - 2T_\alpha)$ by $(58/3 - 2T_\alpha)$.

The structure of the coefficients Λ_{ik} is that of an expansion in some effective Knudsen number for the mixture. This is especially convenient when we turn to the case of a pure (single-component) gas, for which the Knudsen number is defined as $\mathrm{Kn} = \eta/p_0\beta^{1/2}R$. Corresponding expressions for the coefficients Λ_{mm}, $\Lambda_{mq} = \Lambda_{qm}$, and Λ_{qq} for a pure gas, in the form of expansions up to terms proportional to the square of the Knudsen number, are presented in [6]. In the case of a binary mixture these coefficients depend in a complex manner on a series of parameters characterizing the mixture. In the range of low Knudsen numbers, by limiting the expansion to terms linear in Knudsen number, we arrive at results corresponding to the so-called "viscous flow regime with slip." Then

$$\Lambda_{mm} = T_0\eta^{-1}\left(\frac{R^2}{8} + \zeta\frac{R}{2}\right); \quad \Lambda_{mq} = \Lambda_{qm} = -T_0 A_{\mathrm{T}}; \ \Lambda_{qq} = T_0^2\lambda'.$$

Here λ' is the thermal conductivity coefficient, while ζ and A_T are the viscous and thermal slip coefficients for the binary mixture, which are given by

$$\zeta = \frac{\sqrt{\pi}\,\varepsilon^2\eta}{8} + \sum_{\alpha} \beta_\alpha^{-1/2} \frac{(2 - \varkappa_\alpha)\,\eta_\alpha^2}{\sqrt{\pi}\, p_0 y_\alpha \eta},$$

$$A_{\mathrm{T}} = [D_{12}]_2[\alpha_{\mathrm{T}}]_1 \left(T_1\varepsilon + \frac{\varkappa_1\eta_1}{2\eta} - y_1\right) + \frac{T_0}{5p_0} \sum_{\alpha} \frac{\lambda_\alpha'}{y_\alpha}\left(T_\alpha\varepsilon + \frac{\varkappa_\alpha\eta_\alpha}{\eta}\right),$$

where

$$\varepsilon = \frac{1}{2\eta}\sum_{\alpha} \eta_\alpha (2 - \varkappa_\alpha);$$

$$B = \sum_{\alpha} \varkappa_\alpha \beta_\alpha^{1/2} p_{\alpha 0}; \ T_\alpha = \varkappa_\alpha \beta_\alpha^{1/2} p_{\alpha 0} B^{-1}.$$

The coefficients Λ_{11}, Λ_{1m}, and Λ_{1q} are related to diffusion processes for the mixture flow in the channel. In particular, the difference between the average component velocities in the channel can be expressed as

$$\langle u_{1z} \rangle - \langle u_{2z} \rangle = -\frac{D_{12}}{y_1 y_2}\left(\frac{d}{dz} y_1 + y_1 y_2 \alpha_p \frac{1}{p_0}\frac{dp}{dz} + y_1 y_2 \alpha_T \frac{1}{T_0}\frac{dT}{dz}\right).$$

The diffusion, barodiffusion, and thermodiffusion coefficients are then related to the corresponding coefficients Λ_{ik} as follows:

$$D_{12} = \Lambda_{11} y_1 y_2 p_0 T_0^{-1};\quad D_{12}\alpha_p = \Lambda_{1m} p_0 T_0^{-1};\quad D_{12}\alpha_T = \Lambda_{1q} T_0^{-1}.$$

In the region Kn → 0 the coefficients D_{12} and α_T coincide with the conventional diffusion and thermodiffusion coefficients in a gas mixture. Under these conditions the barodiffusion coefficient proves to be independent of channel geometry and is defined by the expressions presented in [4, 5].

To illustrate the dependence of D_{12}, α_p, α_T on mixture molecular properties, Knudsen number, and the character of molecular scattering on the walls, it is useful to consider a mixture whose components have relatively similar masses and scattering sections ($\Delta m/2m \ll 1$ and $\Delta\sigma/2\sigma \ll 1$), using the solid sphere model for the molecules. Also considering the possibility of a small difference in the molecular reflection coefficients on the walls, after simplifying the corresponding expressions (for a mixture with $y_1 = y_2 = 0.5$) we obtain

$$D_{12} = [D_{12}]_2 (1 - 0.6523\varkappa \mathrm{Kn}), \tag{23}$$

$$D_{12}\alpha_p = [D_{12}]_2 \Big\{[1.1441 + \varkappa(0.1303 - 1.2346\mathrm{Kn})]\frac{\Delta m}{2m} + [0.0678 - \varkappa(0.6653 - 1.1712\mathrm{Kn})]\frac{\Delta\sigma}{2\sigma} + [1.9322 - \varkappa(0.0339 - 0.1529\mathrm{Kn})]\frac{\Delta\varkappa}{2\varkappa}\Big\}, \tag{24}$$

$$D_{12}\alpha_T = -[D_{12}]_2\Big[(0.8898 - 1.1114\varkappa\mathrm{Kn})\frac{\Delta m}{2m} + (0.3390 - 0.3086\varkappa\mathrm{Kn})\frac{\Delta\sigma}{2\sigma} + 0.3443\varkappa\mathrm{Kn}\frac{\Delta\varkappa}{2\varkappa}\Big], \tag{25}$$

where

$$\mathrm{Kn} = \frac{5\sqrt{\pi}}{8}(\sqrt{2}\,\pi n_0 \sigma^2 R)^{-1}.$$

As would be expected, differences between the values of these coefficients and the corresponding coefficients for a planar channel [4] occur only in terms which are of the highest order in the expansion in Knudsen number as compared to the conventional kinetic coefficients valid at Kn ≪ 1.

NOTATION

$k_{\alpha,T}$, relative partial pressure and temperature gradients; r, φ, z, variables in cylindrical coordinate system; $u_{\alpha z}$, hydrodynamic velocity of α-component mixture; $p_{\alpha rz}$, partial viscous stress tensor; $h_{\alpha z}$, reduced partial thermal flux; $s_{\alpha ijk}$, $\Pi_{\alpha ijkl}$, third- and fourth-order partial moments; R, channel radius; $\langle h_z\rangle$, $\langle u_{\alpha z}\rangle$, $\langle u_{mz}\rangle$, reduced thermal flux, hydrodynamic velocity of α-component mixture, and mean-molecular mixture velocity averaged over channel section; Λ_{ik}, kinetic coefficients; $\varkappa_\alpha$, fraction of molecules of α-component mixture experiencing diffuse reflection on wall; D_{12}, α_p, α_T, diffusion, barodiffusion, and thermodiffusion coefficients; Kn, Knudsen number; ζ, A_T, viscous and thermal slip coefficients of binary mixture; λ', thermal conductivity coefficient.

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WAVE PROPERTIES AND SHEAR STRESS OF A TURBULENT BOUNDARY LAYER

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The wave theory of turbulence [1-3] is applied to the problem of a turbulent boundary layer near a planar wall. Preliminary results earlier published have been refined.

In a turbulent flow the statistical ensemble state of large-scale vortices is described by the equation [2]

$$ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2\rho}\nabla^2\psi,\ i = \sqrt{-1}, \tag{1}$$

where ∇^2 is the Laplacian. The special representation $\psi = a \exp ib$ makes it possible to obtain from (1) an expression for the energy of motion $h\omega$ and an equation for the probability flow a^2 for stationary turbulence ($\partial a^2/\partial t = 0$):

$$\omega = \frac{1}{2}\rho U^2 - \frac{h^2}{2\rho}\frac{\nabla^2 a}{a}, \tag{2}$$

$$h \operatorname{div}(a^2 \operatorname{grad} b) = 0. \tag{3}$$

In this case $h|\operatorname{grad} b| = \rho U$, $\omega = \partial b/\partial t$. The negative term on the right-hand side of Eq. (2) reflects the statistical aspect of vortex-particle interactions, and equals the fluctuation energy $a^2\rho U^2/2$. Thus,

$$\nabla^2 a + |\operatorname{grad} b|^2 a^3 = 0 \tag{4}$$

and, besides, $h\omega = (1 + a^2)\rho U^2/2$. Since $h = \rho U|\operatorname{grad} b|^{-1}$, it follows that

$$\omega = \frac{1+a^2}{2} U|\operatorname{grad} b|. \tag{5}$$

The amplitude a coincides with the local turbulence intensity u'/U, where u' is the fluctuation in translational velocity. The representation of kinetic properties of vortex-particles in terms of wave characteristics implies that the individual motion of vortices is expressed in terms of statistical ensemble properties, thus forming a set of vortex-particles. The probability distribution of the amplitude a is such that in the region of wave existence

$$\int a^2 db = 1. \tag{6}$$

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